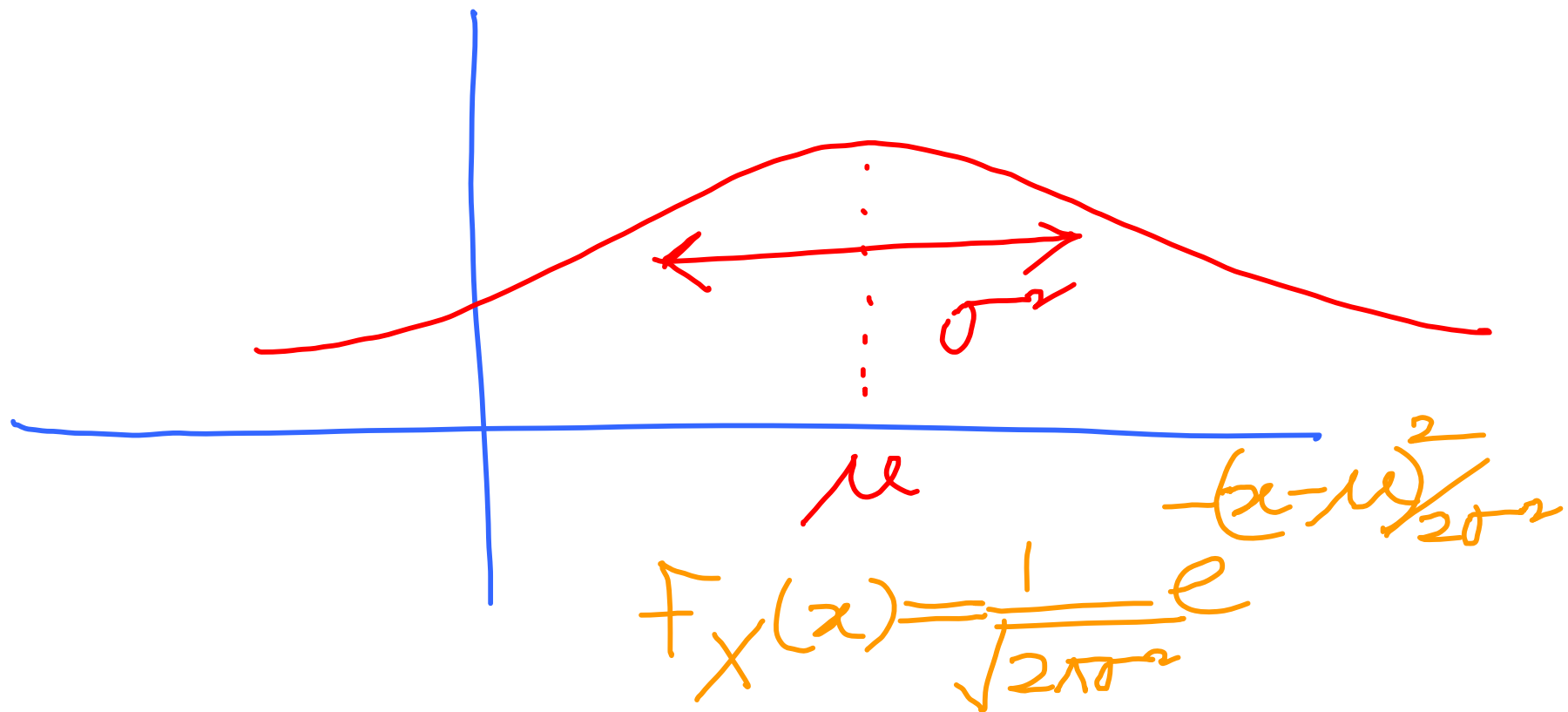


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DIFFERENTIAL ENTROPY OF GAUSSIAN SOURCE

one of most
commonly occurring
& Practically
relevant sources.



$$h(X) = \int_{-\infty}^{\infty} f_X(x) \log_2 \left(\frac{1}{f_X(x)} \right) dx$$

Simplify

$$\frac{1}{f_X(x)} = \frac{1}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$$= \sqrt{2\pi\sigma^2} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\log_2 \left(\frac{1}{f_X(x)} \right) = \ln \left(\frac{1}{f_X(x)} \right) \cdot \log_2 e$$

$$= \log_2 e \cdot \ln \cdot \sqrt{2\pi\sigma^2} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

$$= \log_2 e \left\{ \frac{1}{2} \ln 2\pi\sigma^2 + \frac{1}{2\sigma^2} (x-\mu)^2 \right\}$$

$$\log_2\left(\frac{1}{f_X(x)}\right) = \log_2 \left\{ \frac{1}{2} \ln 2\pi\sigma^2 + \frac{1}{2\sigma^2} (x-\mu)^2 \right\}$$

Substitute this in
expression for
Differential Entropy.

$$h(X) = \int_{-\infty}^{\infty} f_X(x) \log_2 \left\{ \frac{1}{2} \ln 2\pi\sigma^2 + \frac{(x-\mu)^2}{2\sigma^2} \right\} dx$$

$$= \log_2 e \int_{-\infty}^{\infty} f_X(x) \cdot \underbrace{\frac{1}{2} \ln(2\pi\sigma^2)}_{\text{constant}} dx$$

$$+ \log_2 e \int_{-\infty}^{\infty} f_X(x) \cdot \frac{(x-\mu)^2}{\underbrace{2\sigma^2}_{\text{constant}}} dx$$

$$= \log_2 e \cdot \left(\frac{1}{2} \ln 2\pi\sigma^2 \right) \underbrace{\int_{-\infty}^{\infty} f_X(x) dx}_{=1 \text{ Property of PDF}}$$

$$+ \log_2 e \cdot \frac{1}{2\sigma^2} \underbrace{\int_{-\infty}^{\infty} (x-\mu)^2 f_X(x) dx}_{= \text{variance } \sigma^2}$$

$$= \log_2 e \times \frac{1}{2} \ln(2\pi\sigma^2) \times 1$$

$$+ \log_2 e \times \frac{1}{2\sigma^2} \times \sigma^2$$

$$= \frac{1}{2} \log_2(2\pi\sigma^2) + \frac{1}{2} \log_2 e$$

$$h(x) = \frac{1}{2} \log_2(2\pi\sigma^2 e)$$

increases with variance.

DE of Gaussian
Source with mean μ
var $= \sigma^2$

Does NOT depend on mean μ
Depends only on variance!

